Techniques for Fisheye Lens Calibration using a Minimal Number of Measurements

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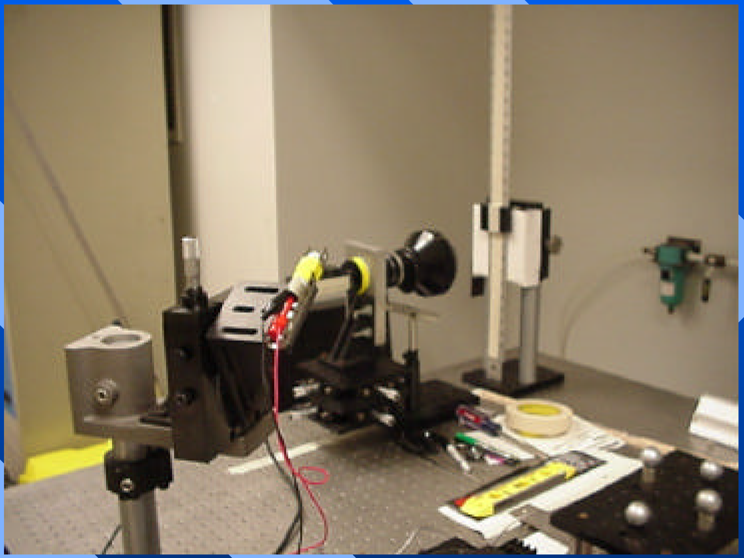
What is Fisheye Distortion?

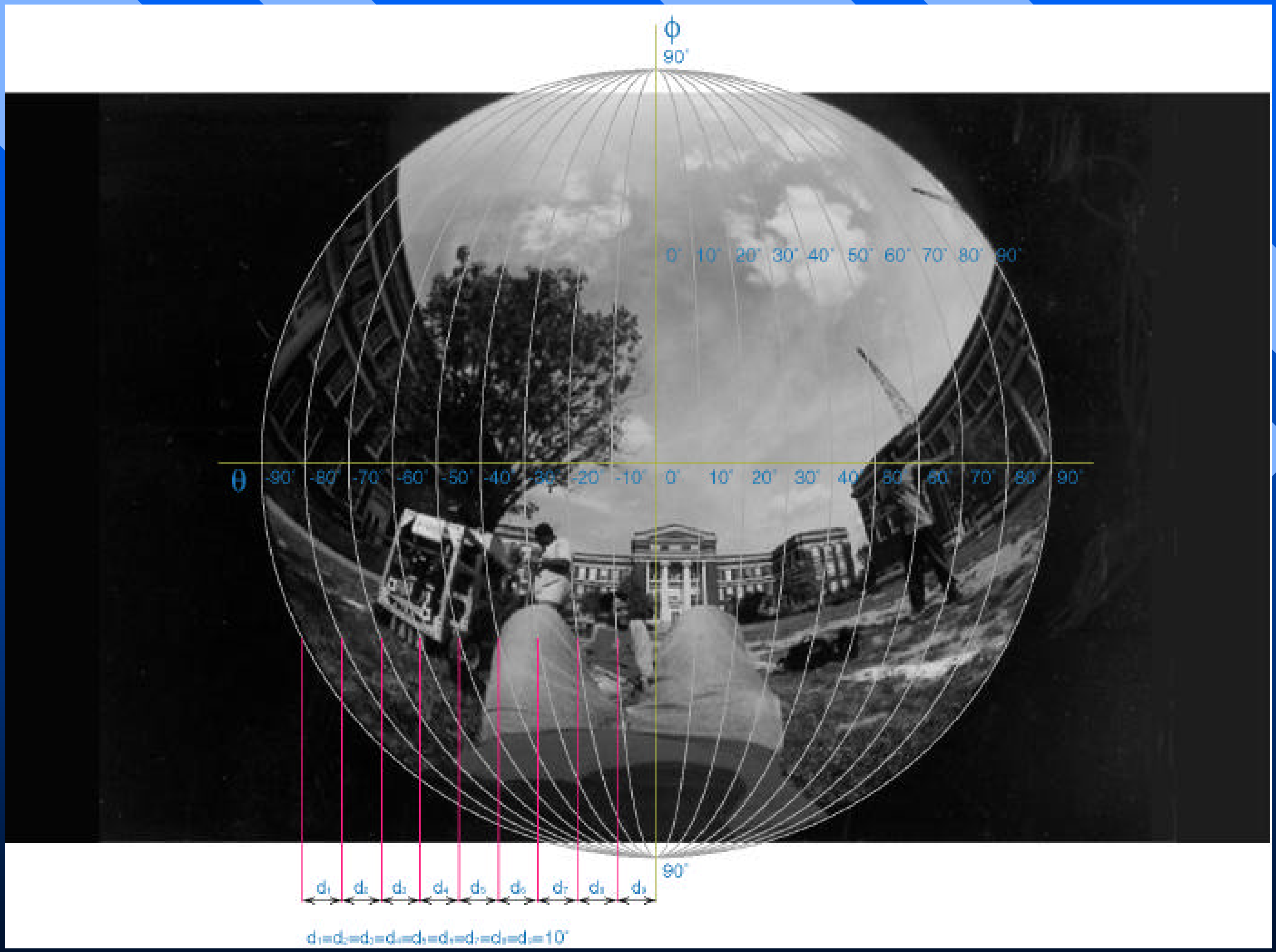
A view through a Nikon Nikkor ‘Fisheye’ lens

Apparatus for Study

- Optical Bench
- Off the Shelf Laser Pointer
- Long Metal Ruler
- Nikon Nikkor 8mm 1:2.8 243304 lens
- ISCAN RK-446-R Automatic Video Tracking system Standard PC
- MTI CCD-725 Camera

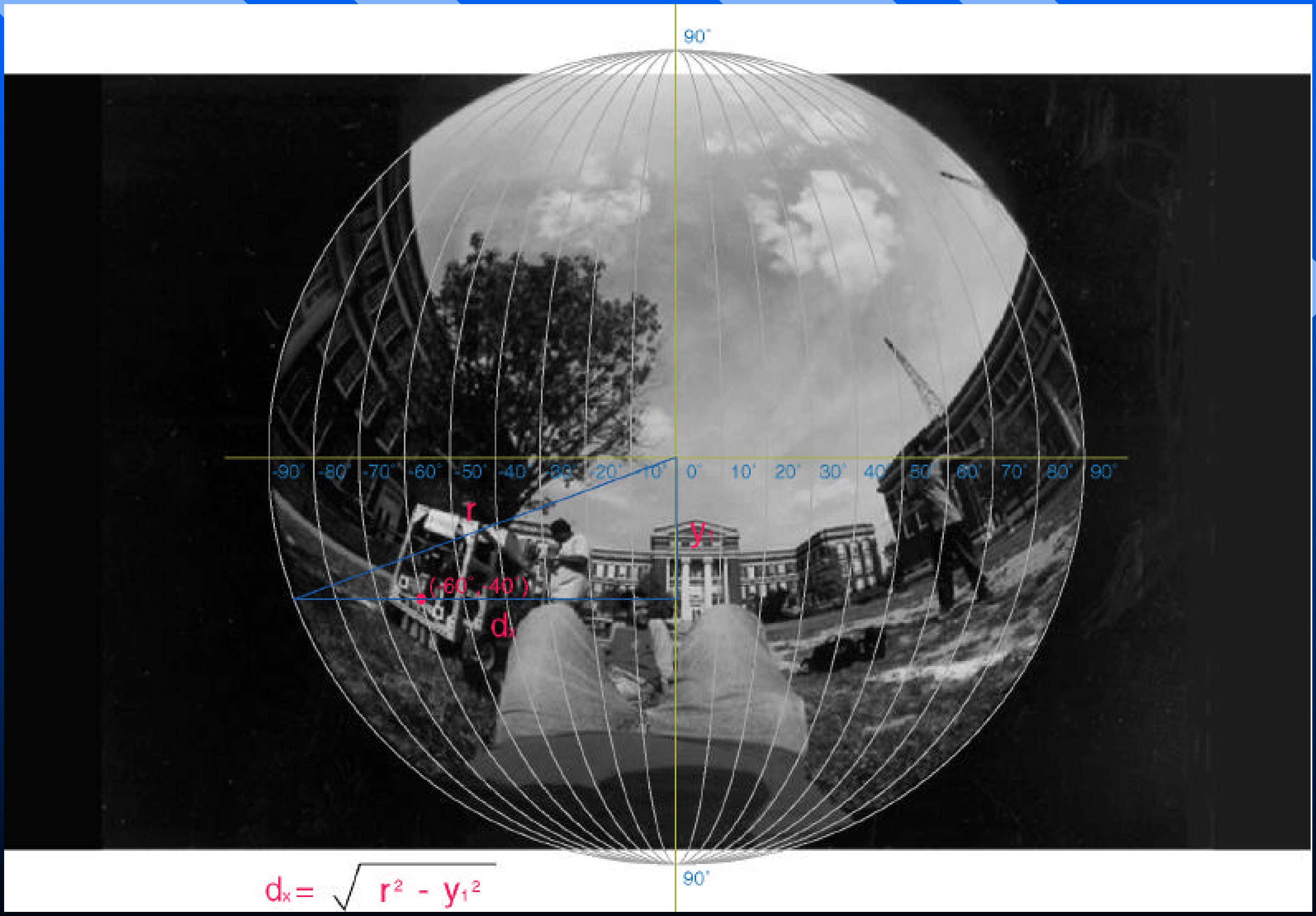


General Distortion Model



The curved lines show how a target can fall at the same angle from the lens if it falls on the same line

The distance between the edge of the viewable area and the x center can be found using the Pythagorean theorem



General Distortion is Solved With This Equation

$$\theta = u\left(\frac{x_d}{r_x\sqrt{1-\left(\frac{y_d}{r_y}\right)^2}}\right)$$

θ = the angle to target, in this case in degrees

u = units, in this case it is set to 90 to represent 90 degrees. You can set u to the number of pixels in the radius for pixel to pixel conversion

r_x = the pixel radius in x, this is usually equal to r_y [1]

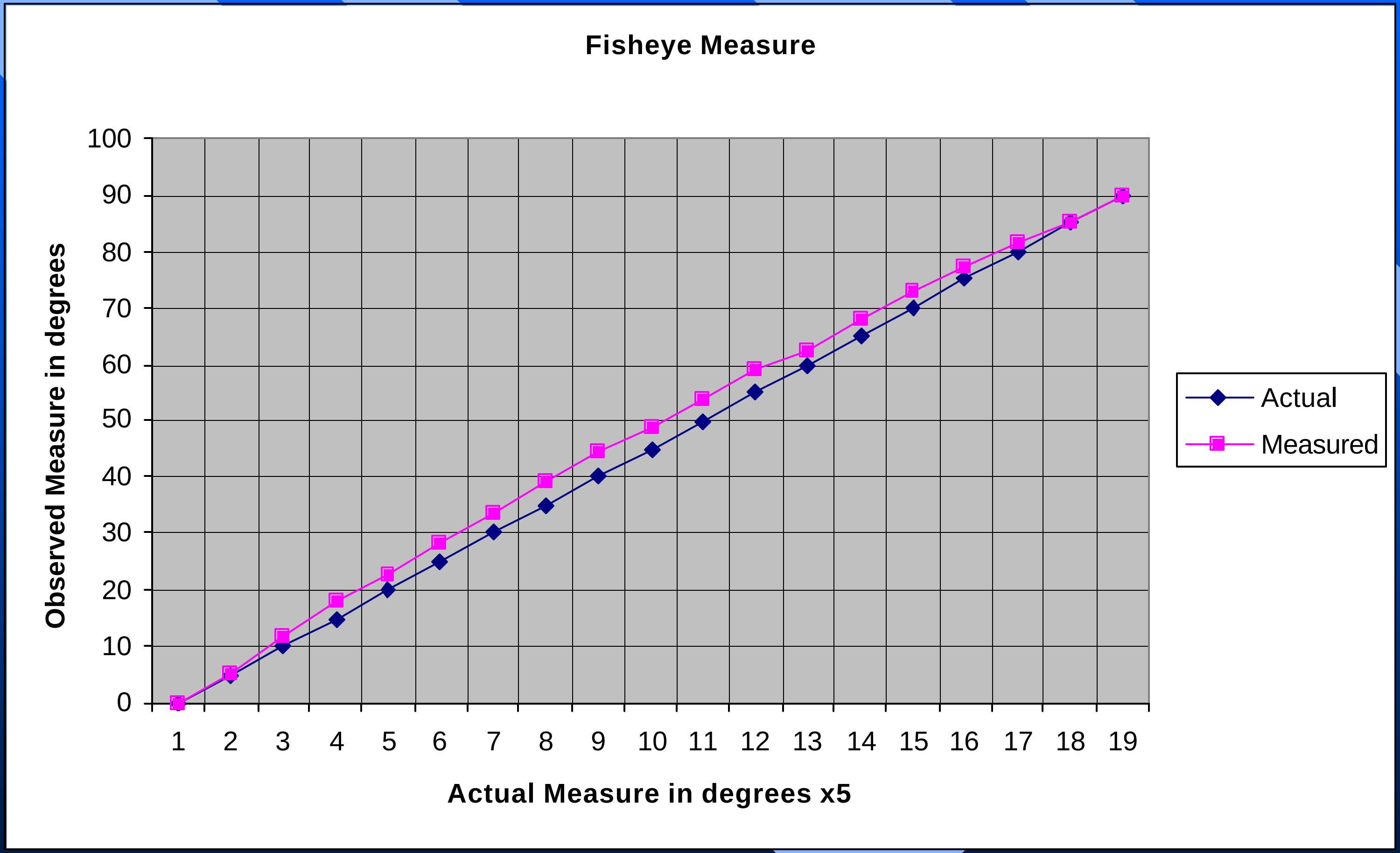
x_d = the distance from the x center of the target in pixels

r_y = the pixel radius in y, this is usually equal to r_x [2]

y_d = the distance from the y center of the target in pixels

[1] The ISCAN device uses rectangular pixels such that the pixel radius in y and x are not equal. This may not be the same in all cases.

Special Distortion Model



This graph shows how the special distortion skews the observed angle to the target as opposed to the real angle.

Special Distortion Solved using Circular Regression

$$h(2\sum_{i=1}^{N} x_i^2) + k(2\sum_{i=1}^{N} x_i y_i) + c(2\sum_{i=1}^{N} x_i) = \sum_{i=1}^{N} x_i z_i^2$$

$$h(2\sum_{i=1}^{N} x_i y_i) + k(2\sum_{i=1}^{N} y_i^2) + c(2\sum_{i=1}^{N} y_i) = \sum_{i=1}^{N} y_i z_i^2$$

$$h(2\sum_{i=1}^{N} x_i) + k(2\sum_{i=1}^{N} y_i) + cN = \sum_{i=1}^{N} z_i^2$$

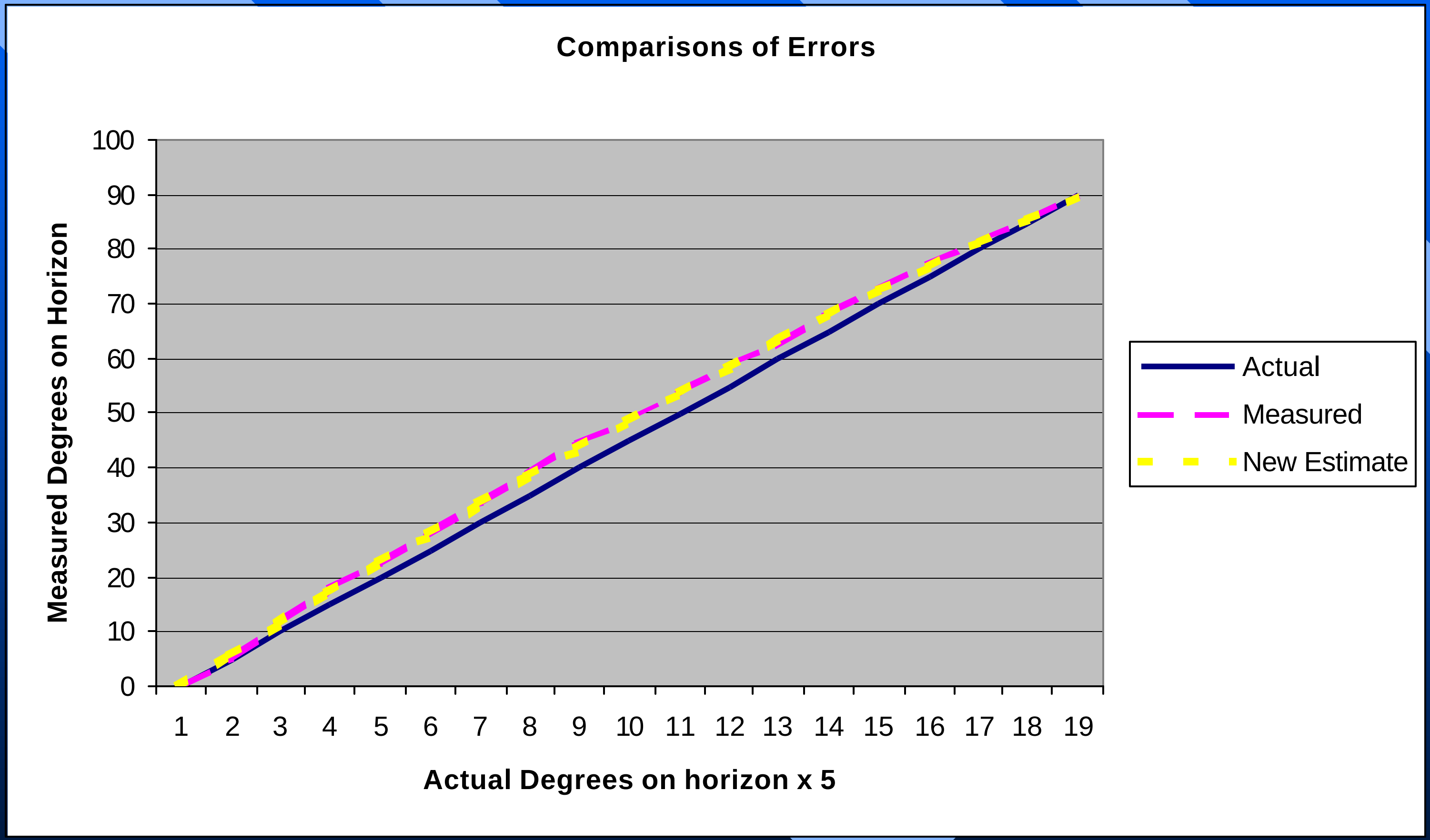
$$C = \begin{bmatrix} \sum_{i=1}^{N} x_i z_i^2 \\ \sum_{i=1}^{N} y_i z_i^2 \\ \sum_{i=1}^{N} z_i^2 \end{bmatrix} \quad B = \begin{bmatrix} h \\ k \\ c \end{bmatrix} \quad A = \begin{bmatrix} 2\sum_{i=1}^{N} x_i^2 & 2\sum_{i=1}^{N} x_i y_i & \sum_{i=1}^{N} x_i \\ 2\sum_{i=1}^{N} x_i y_i & 2\sum_{i=1}^{N} y_i^2 & \sum_{i=1}^{N} y_i \\ 2\sum_{i=1}^{N} x_i & 2\sum_{i=1}^{N} y_i & N \end{bmatrix}$$

$$z_i^2 = x_i^2 + y_i^2$$

For a given set of N data points, a matrix inverse program may be written to solve for h,k and c. The radius, r, may then be found using:

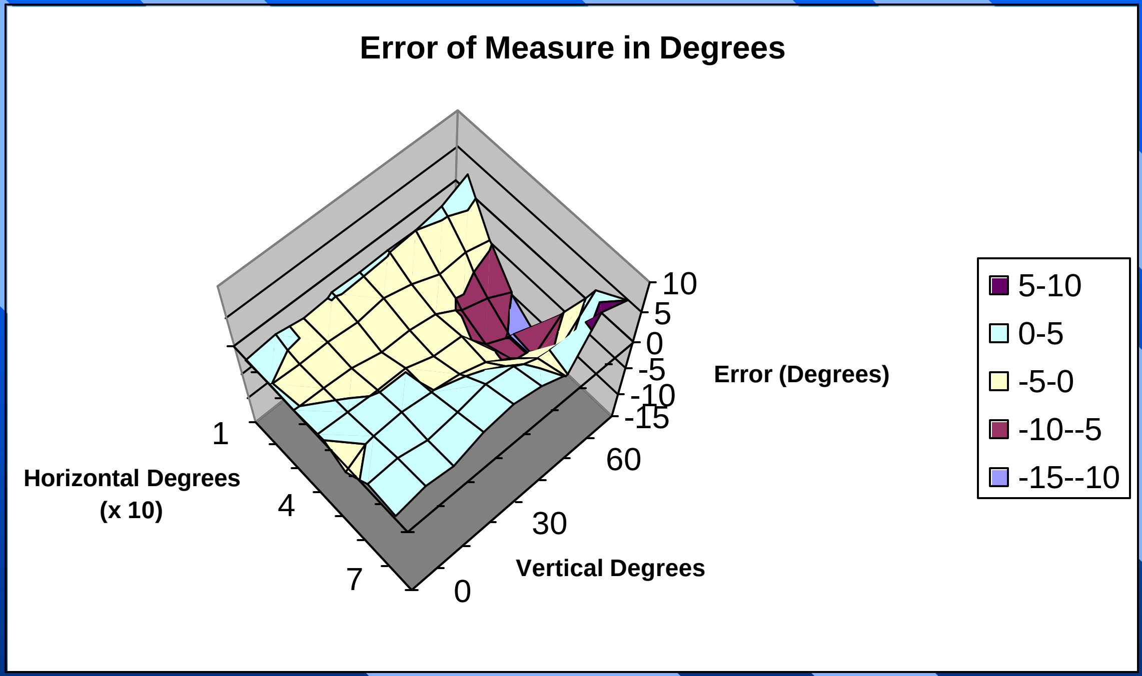
$$r = \sqrt{c + h^2 + k^2}$$

Circular Regression Fits Special Distortion Well



$$\theta_{real} = -(\sqrt{(-(\theta_{obs} - k)^2 + r^2})-h)$$

Experimental Results



		Horizontal Degrees to Target							Degrees per
		0	15	30	45	60	75	85	Pixel
Vertical	0	0	0.02	0.07	0.38	-1.8	1.2	0.6	0.5625
Degrees	10	0.45	-0.36	-0.266	0.226	0.593	1.4	1.48	0.567218
To Target	20	-0.5	-0.52	-0.536	-0.089	0.82	0.48	0.63	0.580566
	30	0.099	-0.91	-1.774	-0.116	0.087	1.26	2.44	0.607108
	40	0.087	-0.52	-1.72	-2.21	-0.82	1.944	3.133	0.647396
	50	0.075	-1.68	-2.64	-2.587	-2.93	0.9169	2.26	0.712554
	60	0	-3.35	-5.69	-7.886	-6.14	-1.52	-0.27	0.802088
	70	0.87	-3.19	-7.49	-10.35	-10.27	-2.62	7	0.987185
	80	2.56	-4.52	-9.8	-12.67	-5.79	2.5	5	1.537721

Error Compared With Pixel Size

error at 45° divided by pixel size			avg error divided by pixel size		
0.675556			1.033651		
0.398436			1.202611		
0.153299			0.879683		
0.19107			1.573266		
3.413673			2.302409		
3.630602			2.624142		
9.831843			4.427019		
10.48435			6.047496		
8.239466			3.979915		

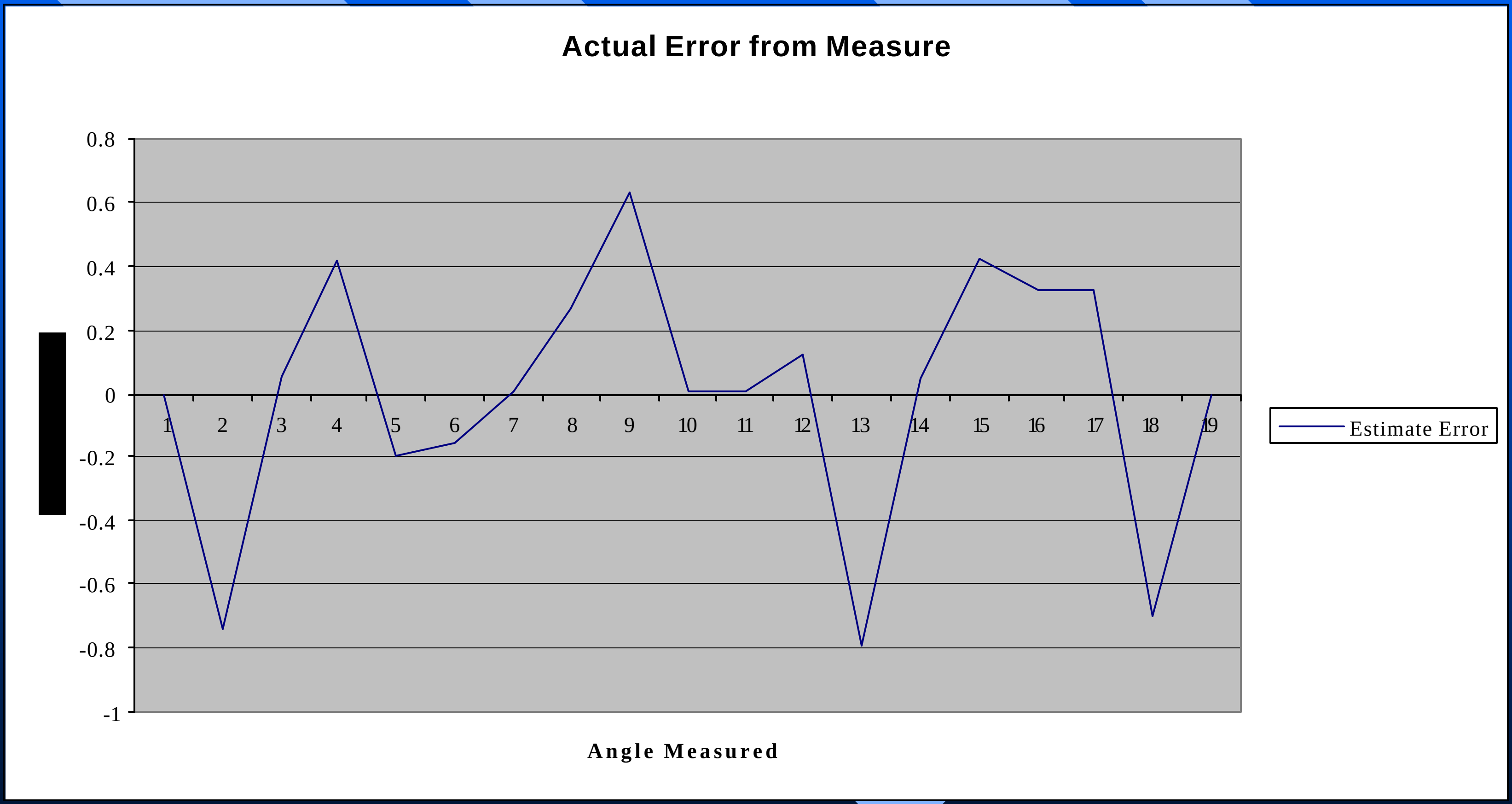
Source of Errors

- Pixels represent greater angle as you move towards 90°
- Laser target was approx two or three pixels in size at horizon
- Possible miscalibration as camera moved for >60° measures
- Rectangular pixels from ISCAN made calibration difficult

Strengths of Circular Regression

- Central Limit Theorem helps Accuracy
- Pick and choose data points
- May allow less accurate techniques
- Can be computed on a TI-92
- Dealing with a circle

Circular Regression Helps in Calibration



Conclusion

- Circular Regression is viable and helpful
- Technique is simple to use
- Technique is viable

Thank You